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Existence of the Gravitomagnetic Interaction

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The point of view expressed in the literature that gravitomagnetism has not yet been observed or measured is not entirely correct. Observations of gravitational phenomena are reviewed in which the gravitomagnetic interaction—a post-Newtonian gravitational force between moving matter—has participated and which has been measured to 1 part in 1000. Gravitomagnetism is shown to be ubiquitous in gravitational phenomena and is a necessary ingredient in the equations of motion, without which the most basic gravitational dynamical effects (including Newtonian gravity) could not be consistently calculated by different inertial observers.

1. INTRODUCTION

In the overview *Physics Through the 1960s*, the National Academy of Sciences (1986) review of opportunities for experimental tests of general relativity, they declare that "At present there is no experimental evidence arguing for or against the existence of the gravitomagnetic effects predicted by general relativity. This fundamental part of the theory remains untested." Similar points of view have been expressed elsewhere in promotion of various experiments designed to "see" gravitomagnetism.

In this paper I make two points on this issue, which together lead to a position contrary to the viewpoint summarized by the above statement.

1. The gravitomagnetic interaction is a consequence of the gravitational vector potential. This vector potential pays a crucial, unavoidable role in gravitation; without the gravitational vector potential the simplest gravitational phenomena—the Newtonian-order Keplerian orbit and the deflection of light by a central body—cannot be consistently calculated in two or more inertial frames of observation. Gravitation without the vector potential is an incomplete, ambiguous theory in the most fundamental sense.

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2. There exists a variety of accurate observations of various post-Newtonian gravitational effects which together measure the gravitational vector potential and whose calculation unavoidably requires the participation of the gravitational vector potential and the resulting gravitomagnetic forces.

In summary, inertial frame "dragging"—both linear accelerative dragging and rotational "Lence-Thirring" dragging—are ubiquitous in gravitational phenomena already observed and measured.

2. COMPLETENESS OF GRAVITATIONAL THEORY

A dynamical theory of gravity, to be complete, must permit any inertial observer to calculate a given phenomenon and obtain a result consistent with the calculated result of another observer in a different inertial frame who analyzes the same phenomenon.

For example, in the rest frame of the sun (or any other body) it is typically assumed that the static, spherically symmetric metric gravitational field is (in isotropic spatial coordinates) at linear order

$$g_{00} = 1 - \frac{2GM}{c^2 r} \tag{1a}$$

$$g_{ij} = -\left(1 + 2\gamma \frac{GM}{c^2 r}\right)\delta_{ij} \tag{1b}$$

$$g_{0i} = 0 \tag{1c}$$

for $0 = ct$, $i, j = x, y, z$. The coefficient γ equals 1 in general relativity. Light deflection is calculated from (1a)-(1c) by employing the null-geodesic principle for a light ray,

$$d\tau^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = 0 \tag{2}$$

which yields the coordinate speed of light

$$c(r) = c_\infty\left[1 - (1+\gamma)\frac{GM}{c^2 r}\right] \tag{3}$$

and a lowest order deflection for a ray of light passing at distance D from the central body:

$$\theta_0 = \int_{-\infty}^{\infty} \frac{\partial c(r)}{\partial D}\, dt = 2(1+\gamma)\frac{GM}{c^2 D} \tag{4}$$

Since the incident and final light rays are observed and measured far from the gravitational source, special relativistic kinematics (the Lorentz transformation) can be used to obtain the deflection angle of the same light ray but observed by someone traveling at speed w in a direction opposite the incident light ray. The final photon 4-vector in the central body rest frame

$$k^{\mu} = k[1, \cos\theta_0, -\sin\theta_0, 0]$$

then transforms in the second inertial frame into

$$k^{\mu'} = k\left[\frac{1+(w/c)\cos\theta_0}{(1-w^2/c^2)^{1/2}}, \frac{\cos\theta_0 + w/c}{(1-w^2/c^2)^{1/2}}, -\sin\theta_0, 0\right]$$

and the new deflection angle is directly read off:

$$\sin\theta_w = (1-w^2/c^2)^{1/2}\sin\theta_0/[1+(w/c)\cos\theta_0]$$

To linear order in w and for small deflection angle $\theta_0(D)$, this becomes

$$\theta_w = \theta_0(1-w/c) \tag{5}$$

This same result, however, must also be calculable directly from the dynamics of gravitational theory by an observer who sees the central body moving to the right at speed w. Suppose this is attempted by one who denies the existence of a gravitational vector potential $g_{0i} \doteq \mathbf{h}$ for the moving source and who again uses the metric field (1a)-(1c). The integral (4) now becomes

$$\theta_w = \int_{-\infty}^{\infty} \frac{(1+\gamma)GMD\,dt}{c\{D^2+[(c-w)t]^2\}^{3/2}} = \theta_0\left(1+\frac{w}{c}\right) \tag{6}$$

which is inconsistent with (5).

If a gravitational vector potential

$$g_{0i} \equiv \mathbf{h} = 2(1+\gamma)\frac{GM\mathbf{w}}{c^3 r} + \sigma\nabla\left(\frac{GM\mathbf{w}\cdot\mathbf{r}}{c^3 r}\right) \tag{7}$$

is added to the metric field for the moving source (σ is an arbitrary gauge coefficient) and the null geodesic principle (2) used to give the modified coordinate speed of light,

$$c'(r) = c\left[1-(1+\gamma)\frac{GM}{c^2 r}\right] + \mathbf{h}\cdot\mathbf{c}$$

then agreement with the light deflection prediction (5) is reached. A Lorentz transformation applied to the static metric (1a)-(1c) obtains, of course, the vector potential (7) (up to the arbitrary gauge term) in the new inertial frame,

$$\begin{aligned} g'_{0i} &= L_0^{\mu} L_i^{\nu} g_{\mu\nu} \\ &= L_0^0 L_i^0 g_{00} + L_0^i L_i^i g_{ii} \\ &\simeq 2(1+\gamma)\frac{GM(w)_i}{c^3 r} \end{aligned}$$

to linear order in w. L_α^β is the Lorentz transformation matrix.

Similarly, consider the calculation of the shape of test body orbits around a central body at the Newtonian order. In the rest frame of the central body the Newtonian-order equation of motion is simply

$$\frac{d^2\mathbf{r}}{dt^2}=-\frac{GM\mathbf{r}}{r^3}$$

with resulting Keplerian ellipses for orbits. The test body equation of motion is obtained from the geodesic principle, which for a single particle is equivalent to an effective Lagrangian

$$L=-\left[g_{\mu\nu}(\mathbf{r},t)\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}\right]^{1/2} \tag{8}$$

If one now attempts to calculate the test body orbits as viewed from an inertial frame which moves at velocity $\mathbf{w}$ relative to the central mass and neglects the presence of vector potential by using the gravitational metric field (1a)–(1c) the equation of motion to linear order in $\mathbf{w}$ becomes

$$\begin{aligned}\frac{d^2\boldsymbol{\rho}}{dt^2}=&-\frac{GM\boldsymbol{\rho}}{\rho^3}+\frac{GM}{c^2\rho^3}(2\mathbf{w}\cdot\boldsymbol{\nu}\boldsymbol{\rho}+\mathbf{w}\cdot\boldsymbol{\rho}\boldsymbol{\nu})\\&+2(1+\gamma)\frac{GM}{c^2\rho^3}(\boldsymbol{\rho}\cdot\boldsymbol{\nu}\mathbf{w}-\mathbf{w}\cdot\boldsymbol{\nu}\boldsymbol{\rho})\end{aligned} \tag{9}$$

in which $\boldsymbol{\rho}=\mathbf{r}-\mathbf{w}t$ and $\boldsymbol{\nu}=d\boldsymbol{\rho}/dt$.

Solution of (9) yields an initial circular orbit which blows itself apart (at least until some higher order terms in the equation of motion stop the runaway),

$$\rho(t)=\rho_0-(1+\gamma)\frac{GM}{c^2\rho_0}w\left(t\cos\omega t-\frac{\sin\omega t}{\omega}\right) \tag{10}$$

ω is the circular orbital frequency. The rate of runaway in (10) depends linearly on the observer's relative motion w.

Nonsense! Such an orbit is clearly not a Keplerian ellipse transported at constant velocity $\mathbf{w}$. However, if the gravitational vector potential (7) is added to the static gravitational metric field (1a)–(1c), it eliminates the driving term in (9) responsible for this orbit runaway, leaving a Lorentz-boosted Kepler orbit as the solution to the equations of motion.

The gravitational vector potential (7) is therefore required for a moving mass source whenever (1a)–(1c) is the linear gravitational metric field for a mass at rest; otherwise, the most elementary gravitational effects cannot consistently be calculated from the general inertial frame of reference.

3. THE NECESSARY GRAVITATIONAL VECTOR POTENTIAL

Will and Nordtvedt (1972) give the necessary structure of the gravitational metric field, and to linear order it is

$$g_{00}=1-2\sum_i \frac{m_i}{r_i}-(2\gamma+1)\sum_i \frac{m_i v_i^2}{c^2 r_i}+\alpha_2\sum_i \frac{m_i}{r_i^3}\left(\frac{w\cdot r_i}{c}\right)^2$$

$$+(\alpha_1-\alpha_2)\sum_i \frac{m_i}{r_i}\left(\frac{w}{c}\right)^2+\alpha_1\sum_i \frac{m_i}{r_i}\frac{\mathbf{w}\cdot\mathbf{v}_i}{c^2} \tag{11a}$$

$$g_{ij}=-\left(1+2\gamma\sum_k \frac{m_k}{r_k}\right)\delta_{ij} \tag{11b}$$

$$g_{0i}\equiv\mathbf{h}=\left(2\gamma+2+\frac{\alpha_1}{2}\right)\sum \frac{m_i}{r_i}\frac{\mathbf{v}_i}{c}+\frac{\alpha_1}{2}\sum \frac{m_i}{r_i}\frac{\mathbf{w}}{c}$$

$$-\left(\frac{1+\alpha_2}{2}\right)\nabla\left(\sum_i \frac{m_i}{r_i}\frac{\mathbf{v}_i\cdot\mathbf{r}_i}{c}\right)-\frac{\alpha_2}{2}\nabla\left(\sum_i \frac{m_i}{r_i}\frac{\mathbf{w}\cdot\mathbf{r}_i}{c}\right) \tag{11c}$$

m_i are mass parameters in gravitational units, $m_i=GM_i/c^2$. The $g_{\mu\nu}$ necessarily includes the vector potential (11c) unless $\alpha_1=\alpha_2=0$ (no preferred inertial frames in gravitational physics) and $\gamma=-1$ (pure scalar gravity). Nature has chosen $\gamma\approx 1$, however. The metric field components (11a)-(11c) depend on a choice of coordinates (gauge), and conservation laws for energy, momentum, and angular momentum are assumed, but these conditions do not affect the discussion here. $\mathbf{w}$ is the observer's inertial frame velocity relative to a "preferred" inertial frame. For $\alpha_1=\alpha_2=0$ the general metric field form is the same in all inertial frames, and there are no "preferred" inertial frame physical effects dependent on $\mathbf{w}$. Note that in the general case a source with $\mathbf{v}_i=-\mathbf{w}$ (a source at rest in the "preferred" inertial frame but being observed from a moving frame) generates a vector potential $\mathbf{h}$ of the form (7), which we found was necessary to make the calculation of basic gravitational effects consistent. It should be emphasized that the metric field form given by (11a)-(11c) results from that condition that we must have a consistent and complete gravitational theory in which observers in any inertial frame may analyze the same physical system and obtain intercomparable, consistent predictions.

Equation (11c) yields a magnetic moment-like vector potential for a spinning mass source

$$\mathbf{h}=\left(1+\gamma+\frac{\alpha_1}{4}\right)\frac{G}{c^3}\frac{\mathbf{J}\times\mathbf{r}}{r^3} \tag{12}$$

$\mathbf{J}$ is the angular momentum of the spinning source. This potential creates the so-called gravitomagnetic effects sought in various experiments. But we already know much about the coefficients γ and α_1 which calibrate this vector potential. γ is known to be 1 to within 1 part in 1000 by the latest radar time-of-flight experiments to Mars planetary landers (Hellings, 1984). α_1 has been constrained by many experiments, but the most recent and tightest bound has been made by using binary pulsar system PSR 1913+16 observational data, which include excellent agreement between general relativity's prediction of gravitational radiation reaction effects and the measured orbital period secular rate of change; this agreement requires α_1 to be less than 10^{-6} (Nordtvedt, 1987). A more modest constraint on α_1 can be made using only solar system observations, in which α_1 is constrained to be less than 10^{-4} (Hellings, 1984).

4. ROLE OF THE GRAVITATIONAL VECTOR POTENTIAL IN PHYSICAL EFFECTS

As has been discussed, the gravitational vector potential is needed to properly calculate any physical effect in gravity, unless one happens to pick a special inertial frame in which all sources are static. This should remind us that it is not particularly fruitful to focus on a particular component of a physical tensor, such as $g_{\mu\nu}$. Components mix among themselves as one transforms from one asymptotic inertial frame to another, and some tensor components can be made zero by choosing special coordinate systems. The invariant aspects (or physical degrees of freedom) of the metric field $g_{\mu\nu}$ are labeled by the PPN coefficients, γ, β, ξ, α_1, α_2, etc., which appear in the several components of the physical tensor (Will and Nordtvedt, 1972).

It may be worthwhile at this point to review several calculations of physical effects in gravity which have been subject to measurement, and in which the gravitational vector potential has played a crucial, unavoidable role because these effects could be viewed from no inertial frame in which all the sources of gravity in the problem were static. From the geodesic-based Lagrangian for individual particles (8) it is straightforward to obtain the contribution of the vector potential to the equation of motion. The result is, in linear order,

$$\delta \mathbf{a} = c\left[\frac{\partial \mathbf{h}}{\partial t} - \mathbf{v} \times (\boldsymbol{\nabla} \times \mathbf{h})\right] \tag{13}$$

As will be shown later, the second acceleration term can be viewed as the "Coriolis" acceleration $2\mathbf{v}\times\boldsymbol{\Omega}$ in an inertial frame being rotationally dragged, and the first term can be interpreted as an accelerative dragging of inertial space. Now (13) is applied to several observations.

A. Suppose one is in the instantaneous rest frame of a celestial body which is being accelerated by an external body. The inductive acceleration term $\partial \mathbf{h}/\partial t$ then makes a significant contribution to the inertial reaction of the body which experiences the external acceleration. From (11c), assuming the body as a whole accelerates at rate $\mathbf{a}$, (13) makes an inertial mass contribution

$$\sum_i M_i \delta \mathbf{a}_i = \left(2\gamma + 2 + \frac{\alpha_1}{2}\right)\frac{G}{c^2}\sum_{ij}\frac{M_i M_j}{r_{ij}}\mathbf{a} - \left(\frac{1+\alpha_2}{2}\right)\frac{G_2}{c^2}\sum_{ij}\frac{M_i M_j}{r_{ij}^3}\mathbf{r}_{ij}\mathbf{r}_{ij}\cdot\mathbf{a} \tag{14}$$

the sum over i, j is over the mass elements of the body. Applied to the earth, (14) makes a contribution to our planet's inertial mass at the level of 3.5×10^{-9} of the total mass of the earth (Nordtvedt, 1968, 1982). The lunar laser ranging observations, however, have constrained the gravitational-to-inertial mass ratio of the earth to be 1 to a part in 10^{12} (C. O. Alley, personal communication; Williams *et al.*, 1976). The very large contribution to inertial mass due to the gravitational vector potential (14) is needed to reach agreement with observation.

B. In the center-of-energy rest frame of the binary pulsar system PSR 1913+16 both bodies, the pulsar and its companion, are in motion because of comparable masses for the two bodies. So even in this special frame of reference the vector potential produced by each body contributes to the motion of the other body. Equation (13) produces a total term to the relative acceleration between the bodies,

$$\begin{aligned}\delta\mathbf{a}_{12} = &-\left(2\gamma+2+\frac{\alpha_1}{2}\right)\frac{GM_1M_2}{c^2(M_1+M_2)}\frac{\mathbf{r}}{r^3}\left[v^2+\frac{2G(M_1+M_2)}{r}\right] \\ &+\left(\frac{1+\alpha_2}{2}\right)\frac{GM_1M_2}{c^2(M_1+M_2)}\frac{1}{r^3}[2\mathbf{v}\mathbf{v}\cdot\mathbf{r}+v^2\mathbf{r}-3(\mathbf{v}\cdot\mathbf{r})^2\mathbf{r}]\end{aligned} \tag{15}$$

In the absence of (15) there would be an additional (anomalous) precession of periastron of the orbit in amount (per revolution)

$$\delta\theta = 15\pi\frac{GM_1M_2}{c^2(M_1+M_2)a_0}\frac{1}{1-e^2}$$

a_0 is the orbit semimajor axis and e is the orbital eccentricity. The above precession amounts to 10.5 arc-deg/year for the binary pulsar; the observed total precession of periastron is 4.2 ± 0.01 arc-deg/year (Weisberg and Taylor, 1984). It should then be asked: What is so different between the gravitomagnetic force acting between two bodies traveling in orbits around their common center of energy, and the gravitomagnetic force predicted to exist between two separated spinning masses and for which it is declared that there is no evidence?

5. DRAGGING OF INERTIAL FRAMES AND MACH'S IDEAS

What seems to have especially caught the interest of physicists in searching for the spin–spin interaction in gravity is that this would seem to be a manifestation of ideas of Mach, who a century ago believed that inertia was caused, in some sense, by the universe's matter distribution. Lense and Thirring later showed that, indeed, in general relativity rotating matter would drag the inertial frame around at a slow rate which fell off with distance from the rotating matter,

$$\boldsymbol{\Omega} = \frac{G}{c^3}\left(\frac{\mathbf{J} - 3\mathbf{J}\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}}{r^3}\right) \tag{16}$$

$\mathbf{J}$ is the angular momentum of the spinning body and $\mathbf{r}$ is the distance to the point of space in question, $\boldsymbol{\Omega}(\mathbf{r})$ is the rotation rate and rotation axis for the inertial space at that point of space which is induced by the spinning source. Equation (16) follows from (12) with choice of PPN coefficients appropriate to general relativity, and the identification

$$\boldsymbol{\Omega} = -\frac{c}{2}\boldsymbol{\nabla}\times\mathbf{h}$$

Looking at the general case, one can ask what is the complete effect of the gravitational vector potential in dragging inertial frames? This question can be addressed by calculating the contribution of $\mathbf{h}$ in establishing the geodesic coordinate frames (inertial frames). The general formula

$$[x^\gamma - x^\gamma_{(0)}]' = [x^\gamma - x^\gamma_{(0)}] + \tfrac{1}{2}\Gamma^\gamma_{\alpha\beta}[x^\alpha - x^\alpha_{(0)}][x^\beta - x^\beta_{(0)}] \tag{17}$$

in which $\Gamma^\gamma_{\alpha\beta}$ are the Christoffel symbols produced from first derivatives of the gravitational metric field, gives the transformation from original space-time coordinates x^γ to inertial (geodesic) coordinates $x^{\gamma'}$ in the vicinity of any chosen space-time point $x^\gamma(0)$. Examining solely the vector potential (g_{0i}) contribution to (17) yields

$$[\mathbf{r} - \mathbf{r}_{(0)}]' = [\mathbf{r} - \mathbf{r}_{(0)}] - c\left[\frac{1}{2}\frac{\partial\mathbf{h}}{\partial t}(t - t_0)^2 + \left(\frac{\boldsymbol{\nabla}\times\mathbf{h}}{2}\right)\times(\mathbf{r} - \mathbf{r}_{(0)})(t - t_0)\right] \tag{18}$$

The gravitational vector potential produces in this general case a "dragging" of inertial space at each locality with both an acceleration of the inertial frame at rate

$$\mathbf{a}(\mathbf{r}, t) = -c\,\partial\mathbf{h}/\partial t \tag{19a}$$

and a rotation of the inertial frame at angular rate and axis

$$\boldsymbol{\Omega}(\mathbf{r}, t) = -\tfrac{1}{2}c\boldsymbol{\nabla}\times\mathbf{h} \tag{19b}$$

If we return to the problem of light deflection by a body moving at speed w and employ the vector potential given by (7), we find that (19a) gives no contribution to the light ray deflection; however, (19b) produces a rotational dragging of inertial frames at a rate

$$\Omega(r, t) = (1+\gamma)\frac{GMDw}{c^2}\frac{1}{|\mathbf{r}-\mathbf{w}t|^3}$$

and in a counterclockwise sense. The time integral of this rotation rate over the entire trajectory of the light ray produces the total deflection or rotation angle

$$\delta\theta = -\frac{2w}{c}\theta_0$$

which is what is needed to obtain agreement with (5) as discussed in Section 2.

The periastron precession of the binary pulsar orbit discussed previously received contributions of inertial frame dragging from both (19a) and (19b). The situation can be viewed this way; part of the motion of the two bodies in the binary pulsar results from the "Coriolis" acceleration that each body experiences because the motion of the other body is producing rotational dragging of the inertial frame at the locality of each body in question.

Finally, the accelerated celestial body mentioned previously drags the inertial frames through (19a), with the resulting acceleration of inertial space being

$$\delta\mathbf{a}(\mathbf{r}, t) = -\left(2+2\gamma+\frac{\alpha_1}{2}\right)\frac{U(\mathbf{r}, t)}{c^2}\mathbf{a}$$

in which $U(\mathbf{r})$ is the Newtonian potential function of that body's mass distribution and $\mathbf{a}$ is the body's acceleration.

6. CONCLUSION

The gravitomagnetic interaction—the post-Newtonian gravitational interaction between moving masses—has been observed and measured in a number of different phenomena. The strength of this interaction is now known to an accuracy of 1 part in 1000. The gravitomagnetic interaction is also required in order to have a complete and consistent theory of gravity at all: even static source gravitational effects when viewed in another inertial frame require the gravitomagnetic interaction in order for basic consistency of a theory's equations of motion. Just as in electromagnetic theory, there is no absolute separation of "electric" and "magnetic" effects; such a division is inertial frame dependent.

I believe there are many excellent reasons to develop experiments to see additional gravitomagnetic effects, or, for that matter, to see any other new post-Newtonian gravitational effects, even though all PPN metric coefficients at the first post-Newtonian level have been measured to accuracy of 10^{-3} or better. Redundant measurements of post-Newtonian gravity test the very consistency of the idea that there is a unique PPN metric gravitational field which is sufficient to calculate the outcome of any first post-Newtonian gravitational effect. Failure to confirm this when observing new phenomena would require a revolutionary, not minor, change in our theoretical foundations of gravitation.

But the point of view expressed by the National Academy of Science statement quoted earlier in this paper and repeated throughout the literature is erroneous; there clearly is much experimental evidence for the gravitomagnetic interaction.

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